Estimation of Space-wise and Time-wise Varying Heat Transfer Coefficients for an Impinging Turbulent Round Jet

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ABSTRACT

An inverse heat conduction problem is solved to estimate the space-wise and time-wise varying heat transfer coefficient for a turbulent round jet impinging onto a confined circular flat plate. Two distinct regions are observed: the impingement region and the wall-jet region. The estimation, based on transient wall temperature measurements, captures the radial and time-wise variation of the heat transfer coefficient between the cold impinging jet and the heated circular flat plate. The direct problem is solved through a finite difference method. The inverse heat conduction problem of function estimation is solved by the conjugate gradient method with an adjoint equation.

NOMENCLATURE

h	heat transfer coefficient
k	thermal conductivity
q_w	surface heat flux
R	radius of the plate
T	temperature
t	time
r	radial coordinate
z	longitudinal coordinate
J	squared residue functional
P	direction of descent
Y	measured temperature
λ	adjoint function
σ	thermal diffusivity
γ	conjugate coefficient
α	thermal diffusivity
β	step size in conjugate grad. meth.
Superscripts	
n	level in time discretization
$\overline{n+1}$	intermediate level in time discret.
Subscripts	
0	initial condition
av	averaged variable
f	final

INTRODUCTION

Inverse heat transfer problems have been investigated extensively in recent years with a view to important applications. These applications, very often involve the estimation of thermophysical properties of solids, of unknown boundary or initial conditions, of geometrical configurations and of heat source strength [1-6]. As a result, many numerical and analytical techniques have been developed specifically for the solution of inverse heat conduction problems. Typical representatives are the function specification method, the Tikhonov regularisation method, the mollification method, and the Alifanov's iterative regularisation method. In general, these methods of solution reduce their formulations to minimization problems subjected to some stabilization technique. The conjugate gradient method, e.g., is a typical powerful minimization technique, which can be applied to function estimation together with a suitable stopping criterion to obtain stable solutions for the inverse problem.

Most inverse boundary problems in heat conduction are concerned with the estimation of boundary heat flux. The estimation of time varying heat transfer coefficient, on the other hand, has received less attention.

The problem of particular interest in this work is to solve an inverse heat conduction problem for the estimation of the space-wise and time-wise varying heat transfer coefficient of a turbulent round jet impinging onto a circular flat plate. The heat transfer coefficient is determined from simulated temperature data on the top of the surface.

The inverse heat conduction problem for the estimation of time varying heat transfer coefficients is solved by applying Alifanov's iterative regularisation method. In this approach, an optimisation problem is solved in which a squared residue functional is minimised through the conjugate gradient method. A sensitivity problem is solved to determine the step size in the direction of descent and an adjoint problem is solved to determine the gradient of the functional. No prior information is used on the functional form of the heat transfer coefficient variation with time.

A SHORT REVIEW ON THE IMPINGING JET PROBLEM

After some general statements on impinging jets, we will close this section by establishing a connection between the results to be developed in this work and their importance in view of the developments of other authors.

A cold turbulent jet impinging normally onto a heated surface is a very effective means to promote high rates of heat exchange. Thus, this geometrical arrangement has been extensively used in industrial processes that aim to achieve intensive heating, cooling or drying rates. Typical applications are the tempering and shaping of glass, the annealing of plastic and metal sheets, the drying of textile and paper products, and the cooling of electronics instruments. Due to the many applications and to the high complexity of the flow structure resulting from an impinging jet, many recent works have been conducted to understand the heat transfer characteristics.

Colucci and Viskanta [7] studied experimentally the effects of nozzle geometry on the local heat transfer coefficients of confined impinging jets. Low nozzle-to-plate gaps were considered in the Reynolds number range of 10,000 to 50,000. The results were compared with similar experiments for unconfined jets. An important conclusion was that the local heat transfer coefficients for confined jets are more sensitive to Reynolds number and nozzle-to-plate gaps than those for unconfined jets.

The heat transfer in the flow of a cold, two-dimensional, vertical liquid jet against a hot, horizontal, surface was given an approximate solution for the velocity and temperature fields by Shu and Wilks [8]. The solution is valid for laminar flows and resorts to the hydrodynamic similarity solution of Watson (see, [8]). The results were compared with a numerical realization of the flow.

Behnia and Durbin [9] reported computations of the flow and thermal fields in an axisymmetric isothermal fully developed turbulent jet, perpendicular to a uniform heat flux flat plate. The V2F model was used in the calculations and, for comparison, computations were also performed with the standard κ-ϵ model. The V2F heat transfer predictions are in agreement with experiments. The κ-ϵ model does not properly resolve the flow features, over-predicts the rate of heat transfer and yields physically unrealistic behaviors.

Wang et al.[10] performed an analytical study of the heat transfer between an axisymmetric impinging jet and a solid surface with non-uniform wall temperature or wall heat flux. The results show that the nonuniformity of wall temperature or wall heat flux has a considerable effect on the stagnation point Nusselt number. In a second study, Wang et al.[11] investigated the heat transfer in the boundary layer region. The results indicated that the Nusselt number for increasing wall temperature or wall heat flux can be considerably higher than that for constant wall temperature or wall heat flux outside the stagnation region.

Unsteady heat transfer caused by a confined impinging jet flow was studied using direct numerical simulation by Chung and Luo [12]. They found that the unsteady heat transfer characteristics are strongly correlated with the vortex dynamics of the jet flow, and unsteady separation induces a secondary maximum and a local minimum of the instantaneous heat transfer along the wall region.

Park et al.[13] have numerically investigated flow and heat transfer characteristics of confined impinging slot jets by using a SIMPLE based segregated streamline upwind Petrov-Galerkin finite element method. Their results for turbulent impinging jets report that the calculated Nusselt number distribution is in good agreement with the experimental data for low Reynolds numbers. However, as the Reynolds number increases, the magnitude and position of the second peak of the Nusselt number disagree more and more with the experimental data.

Thus, we have just seen from the above remarks, that the problem of estimating local convection heat transfer coefficients is central for the impinging jet problem. However, depending on the measuring technique that one is using, this evaluation may be difficult to make due to conjugate effects. That is where the inverse problem technique may come into help. Indeed, through inverse problems the experimentalist will be able to estimate the local heat transfer coefficients from local wall surface temperature measurements, an easily obtained data. The objective of this work becomes then clearer. We strive at developing a numerical tool capable of evaluating the heat tranfer coefficient from simple wall temperature data.

MATHEMATICAL FORMULATION OF THE DIRECT PROBLEM

Consider the transient heat conduction in a thin, circular plate subjected to a constant heat flux on its bottom side and to the convective cooling of a turbulent round jet impinging onto the top side.

The two-dimensional transient heat conduction equation for an homogeneous medium with constant properties in cylindrical coordinates can be written as

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} \right] \qquad (1)$$

in $0 < r < R$, $0 < z < a$, for $t > 0$.

with the following boundary conditions

$$(\partial T/\partial r) = 0, \text{ at } r = 0,\ 0 < z < a,\ t > 0$$
$$(\partial T/\partial r) = 0, \text{ at } r = R,\ 0 < z < a,\ t > 0$$
$$-k(\partial T/\partial z) = q_w, \text{ at } z = 0,\ 0 < r < R,\ t > 0$$
$$-k(\partial T/\partial z) = h(r,t)(T - T_{inf}),$$
$$\text{at } z = a,\ 0 < r < R,\ t > 0$$

and the initial condition

$$T(r, z, 0) = T_0(r, z)$$

in $0 < r < R$, $0 < z < a$, for $t = 0$,

where k is the thermal conductivity of the plate, α is the thermal diffusivity, T_{inf} is the free stream temperature of the impinging jet, a is the thickness of the plate and R its radius.

When the material properties, the initial and boundary conditions are known, the temperature distribution, $T(r, z, t)$ can be determined. Problem (1) is then called the direct problem. On the other hand, if any of these conditions, or a combination of them, is unknown, but, instead, experimentally measured temperatures are available somewhere in the space-time domain, an estimation of the unknown quantities may be attempted. This is known as the inverse heat conduction problem.

Before proceeding directly to the mathematical formulation of the inverse problem, we introduce a lumped-differential formulation of the direct problem, which is reduced to an one-dimensional transient heat conduction problem. We first introduce the spatially averaged temperature along the z direction

$$T_{av}(r, t) = \frac{1}{a} \int_0^a T(r, z, t) dz \qquad (2)$$

Then, Eq.(1) is operated by $(1/a) \int_0^a dz$, to yield

$$\frac{\partial T_{av}(r,t)}{\partial t} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T_{av}(r,t)}{\partial r}\right) + \frac{\alpha}{a}\left(\frac{\partial T}{\partial z}\Big|_{z=a} - \frac{\partial T}{\partial z}\Big|_{z=0}\right) \qquad (3)$$

Now, the boundary conditions to Eq. (1) can be used to give

$$\frac{\partial T_{av}(r,t)}{\partial t} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T_{av}(r,t)}{\partial r}\right) + \frac{\alpha}{ak}\left(-h(r,t)(T(r,a,t) - T_{inf}) + q_w\right) \qquad (4)$$

Eq.(4) is an equivalent integro-differential formulation of the mathematical model, Eq.(1), with no approximations involved.

Supposing that the temperature gradients are sufficiently smooth over the whole spatial solution domain, the classical lumped system analysis (CLSA) is based on the assumption that the boundary temperature can be reasonably well approximated by the average temperature, as $T(r, a, t) \cong T_{av}(r, t)$, which leads to the simple lumped model,

$$\frac{\partial T_{av}(r,t)}{\partial t} = \frac{\alpha}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T_{av}(r,t)}{\partial r}\right) + \frac{\alpha}{ak}\left(-h(r,t)(T_{av}(r,t) - T_{inf}) + q_w\right) \qquad (5)$$

to be solved with the following boundary and initial conditions

$$(\partial T_{av}/\partial r) = 0, \text{ at } r = 0, t > 0$$

$$(\partial T_{av}/\partial r) = 0, \text{ at } r = R, t > 0$$

$$T_{av}(r,0) = T_{av0}(r)$$

in $0 < r < R$, for $t = 0$,

MATHEMATICAL FORMULATION OF THE INVERSE PROBLEM

The purpose of the inverse problem formulation is to estimate the unknown time varying heat transfer coefficient for an impinging turbulent round jet from temperature measurements taken on the top surface of a heated plate. To this end, we will use Alifanov's iterative regularisation method, also known as the conjugate gradient method with an adjoint equation (Su and Silva Neto [12]).

The conjugate gradient method was implemented with an adjoint equation through the following steps: (i) the sensitivity problem, (ii) the adjoint problem and the gradient equation, (iii) the conjugate gradient method of minimisation and (iv) the stopping criterion.

Here, we will provide a brief description of each step, and, then, wil present the solution algorithm enumerating the basic steps.

The sensitivity problem

By introducing a small perturbation on the heat transfer coefficient in the direct problem, that is, $h(r,t) \rightarrow h(r,t) + \Delta h(r,t)$, a small perturbation on the temperature field is expected, $T_{av}(r,t) \rightarrow T_{av}(r,t) + \Delta T_{av}(r,t)$. Substracting from the resulting expression the direct problem, Eqs.(5), and neglecting second order terms, we have

$$\frac{\partial \Delta T(r,t)}{\partial t} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Delta T(r,t)}{\partial r}\right) + \frac{\alpha}{ak}(-h(r,t)\Delta T(r,t) - \Delta h(r,t)(T_{av}(r,t) - T_{inf})) \quad (6)$$

in $0 < r < R$, for $t > 0$.

$$(\partial \Delta T/\partial r) = 0, \text{ at } r = 0, t > 0$$

$$(\partial \Delta T/\partial r) = 0, \text{ at } r = R, t > 0$$

$$\Delta T(r,0) = 0 \text{ in } 0 < r < R, \text{ for } t = 0,$$

The adjoint problem and the gradient equation

The inverse problem is solved as an optimisation problem where we search for the solution $h(r,t)$ that minimises the functional

$$J \equiv J(h(r,t)) = \int_0^{t_f} \sum_{m=1}^{M} (T_{av}(r_m,t) - Y(t))^2 dt, \quad (7)$$

where $T_{av}(r_m,t)$ and $Y(t)$ are the computed and measured temperatures, $m = 1, 2, ..., M$, M being the number of sensors, and $[0, t_f]$ is the interval of time in which experimental data were acquired.

The adjoint problem is developed by defining the Lagrangian

$$J = \int_0^{t_f} \sum_{m=1}^{M} (T_{av}(r_m,t) - Y(t))^2 dt + \int_0^{t_f}\int_0^{R} \lambda(r,t)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_a v(r,t)}{\partial r}\right) + \frac{1}{ak}(-h(r,t)(T_{av}(r,t) - T_{inf}) + q_w) - \frac{1}{\alpha}\frac{\partial T_{av}(r,t)}{\partial t}\right] r dr dt, \quad (8)$$

where $\lambda(r,t)$ is the adjoint function.

Using the same perturbation scheme as applied to the sensitivity problem and neglecting second order terms, we obtain

$$\Delta J = \int_0^{t_f} 2\sum_{m=1}^{M} (T_{av}(r_m,t) - Y(t))\Delta T_{av}(r_m,t) dt + \int_0^{t_f}\int_0^{R} \lambda(r,z,t)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Delta T(r,t)}{\partial r}\right) + \frac{1}{ak}(-h\Delta T(r,t) - \Delta h(r,t)(T_{av}(r,t) - T_{inf})) - \frac{1}{\alpha}\frac{\partial \Delta T(r,z,t)}{\partial t}\right] r dr dt, \quad (9)$$

The adjoint problem is obtained after some manipulation

$$-\frac{1}{\alpha}\frac{\partial\lambda(r,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\lambda(r,z,t)}{\partial r}) + \frac{h}{ak}\lambda + 2\sum_{m=1}^{M}\Big((T(r,z,t)-Y(t)).\,\delta(r-r_m)\delta(z-z_m)\Big), \quad (10)$$

in $0 < r < R$, for $t > 0$.

$$(\partial\lambda/\partial r) = 0, \text{ at } r = 0,\ t > 0$$
$$(\partial\lambda/\partial r) = 0, \text{ at } r = R,\ t > 0$$
$$\lambda = 0 \text{ in } 0 < r < R, \text{ for } t = t_f,$$

The following integral term is left

$$\Delta J = \int_0^{t_f}\int_0^R -\frac{\lambda(r,t)}{ak}(T_{av}(r,t) - T_\infty)\Delta h\,dt. \quad (11)$$

By the definition of gradient, the following relation holds

$$\Delta J = \int_0^{t_f}\int_0^R J'\Big[h(r,t)\Big]\Delta h\,dt \quad (12)$$

A comparison of Eqs.(11) and (12) reveals that the gradient of the functional, $J'(t)$, is given by

$$J'\Big[h(r,t)\Big] = -\frac{\lambda(r,,t)}{ak}(T_{av}(r,t) - T_\infty). \quad (13)$$

The conjugate gradient method of minimisation

The iterative procedure for the estimation of the unknown heat transfer coefficient $h(r,t)$ is given as

$$h^{n+1}(r,t) = h^n(r,t) - \beta^n P^n(r,t), \quad (14)$$

n=0,1,2,..., where the direction of descent $P^n(r,t)$ at step n is defined as

$$P^n(r,t) = J'^n(r,t) + \gamma^n P^{n-1}(r,t), \quad (15)$$

with $\gamma^0 = 0$, and the conjugate coeffient γ^n is given by

$$\gamma^n = \frac{\int_0^{t_f}\int_0^R \Big[J'^n(r,t)\Big]^2 r\,dr\,dt}{\int_0^{t_f}\int_0^R \Big[J'^{n-1}(r,t)\Big]^2 r\,dr\,dt}. \quad (16)$$

The step size β^n is determined by minimising the functional $J[h(r,t)]$ given by Eq.(5), that is

$$\frac{\partial J(h^{n+1})}{\partial\beta^n} = 0. \quad (17)$$

We obtain therefore

$$\beta^n = \frac{\int_0^{t_f}\sum_{i=1}^M (T_{av}(r_m,t) - Y(t))\Delta T(r_m,t)dt}{\int_0^{t_f}\sum_{i=1}^M \Big[\Delta T(r_m,t)\Big]^2 dt}. \quad (18)$$

The stopping criterion

The discrepancy principle is used to establish the criterion for stopping the iterations in the estimation of the heat transfer coefficient, as measurement errors are always present in real applications. Let the standard deviation σ be the same for all measurements, that is

$$T_{av}(r_m, z_m, t) - Y(t) \cong \sigma. \quad (19)$$

Introducing this result into Eq.(3), we have

$$\epsilon^2 = \int_0^{t_f}\sum_{m=1}^M \sigma^2 dt = M\sigma^2 t_f. \quad (20)$$

The iterative procedure is interrupted when

$$J[h(r,t)] < \epsilon^2. \quad (21)$$

The solution algorithm

We now summarize the solution algorithm that implements the iterative procedure as follows:

Step 1. Choose an initial guess $h^0(r,t)$, for example $h^0(r,t) =$ constant;
Step 2. Solve the direct problem, Eqs.(1), to obtain $T_{av}(r,t)$;
Step 3. Solve the adjoint problem, Eqs.(10), to obtain $\lambda(r,t)$;
Step 4. Compute the gradient, $J'(r,t)$, with Eq.(13);
Step 5. Compute the conjugate coefficient, γ^n, with Eq.(16);
Step 6. Compute the direction of descent, $P^n(r,t)$, with Eq.(15);
Step 7. Solve the sensitivity problem, Eqs.(6), with the source term given by $\Delta h(r,t) = P^n(r,t)$, to obtain $\Delta T(r,t)$;
Step 8. Compute the step size, β^n, with Eq.(19);
Step 9. Compute a new estimate, $h^{n+1}(r,t)$, with Eq.(14);
Step 10. Interrupt the iterative procedure if the stopping criterion, Eq.(20), is satisfied; otherwise, go back to Step 2.

RESULTS AND DISCUSSION

A series of numerical simulations were carried out to evaluate the accuracy of the proposed inverse analysis for the estimation of the time-varying heat transfer coefficient in a forced convective flow from an impinging jet over a heated circular plate.

The simulated transient temperature data, $Y_n(t_n)$, $n = 1, 2, ..., n_t$, were generatedby by adding random errors to the temperature data evaluated through the direct problem formulation, $T_n(t_n)$,

$$Y_n = T_n + \sigma e_n, \quad n = 1, 2, ..., n_t,$$

where σ is the standard deviation and e_n is a normally distributed random error. For the normally distributed error, there is a 99% probability for the value of e_n to lie in the range $-2.576 < e_m < 2.576$. In all test cases, 11 temperature points were used to generate numerically the temperature data.

The effects of temperature error readings in the computed temperature for the estimation of space and time-wise varying heat transfer coefficients, was examined. Here, the square-wave variation of the heat transfer coefficient has 0.1 second in duration. Different values of the standard deviation of the measurement errors were used in the simulations. Values of $\sigma = 0.01$, 0.02 and 0.05 with respect to the largest value of simulated temperature data were used. The observation time t_f was 5.0 seconds also.

In Figures 1 to 4 the estimated space-wise varying heat transfer coefficient is shown for $\sigma = 0.0$, 0.01, 0.02 and 0.05. In these Figures, estimates are given for $t = 0.1$, 0.3 and 0.5 t_f. The solid lines represent the values computed by the direct problem whereas the dashed lines represent the estimated data. As can be seen, the heat transfer coefficients are estimated quite well by the inverse solution for the simulated data far away from the impingement region. Near to this point, however, the inverse solution could not capture the correct amplitude of the variation of the heat transfer coefficients.

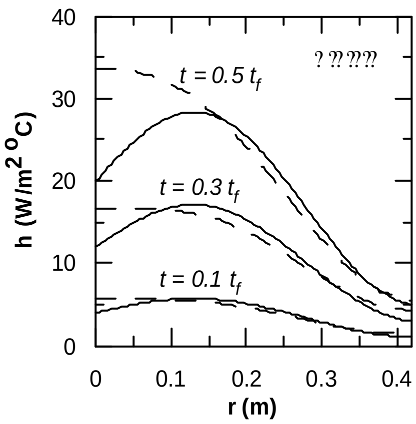

Figure 1: Space varying heat transfer coefficient.

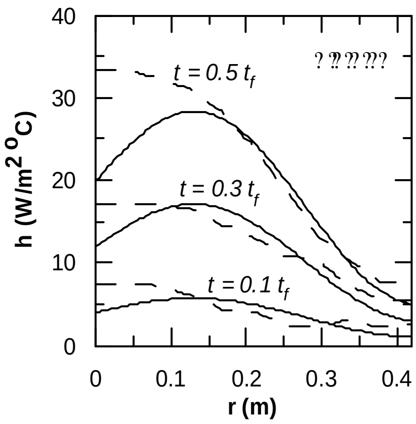

Figure 2: Space varying heat transfer coefficient.

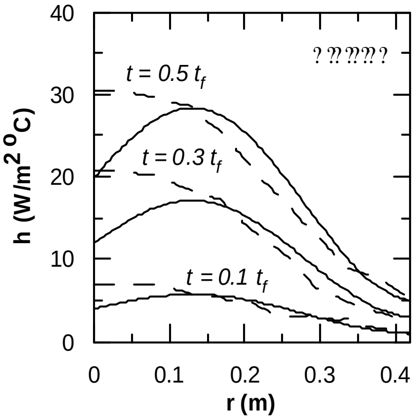

Figure 3: Space varying heat transfer coefficient.

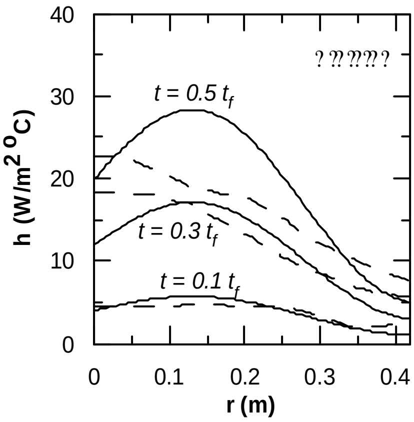

Figure 4: Space varying heat transfer coefficient.

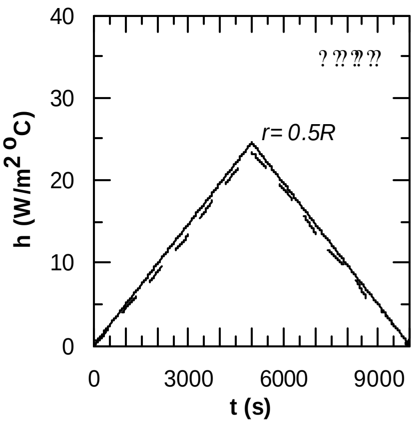

Figure 5: Time varying heat transfer coefficient.

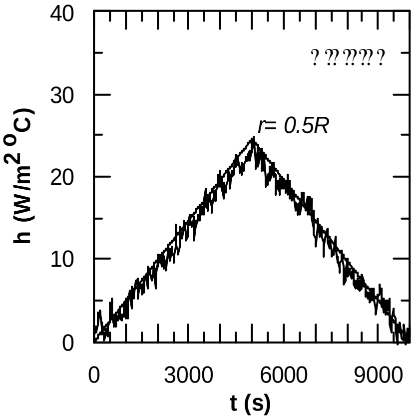

Figure 6: Time varying heat transfer coefficient.

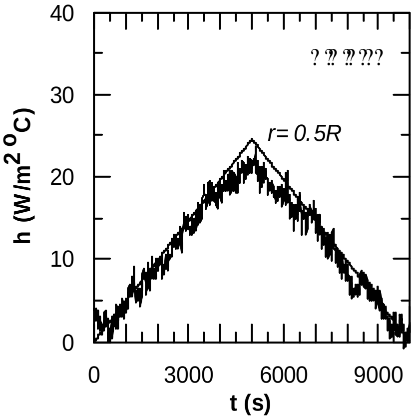

Figure 7: Time varying heat transfer coefficient.

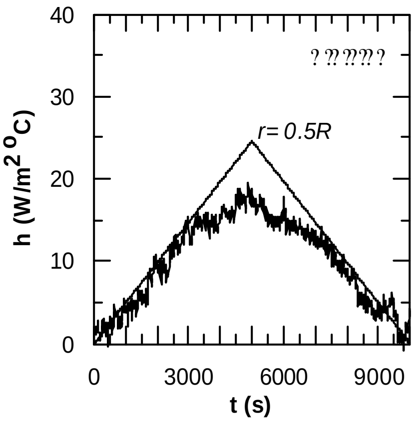

Figure 8: Time varying heat transfer coefficient.

In Figures 5 to 8, the estimated time-wise varying heat transfer coefficients are shown for $\sigma = 0.0$, 0.01, 0.02 and 0.05. The estimates were obtained for one radial position, $r = 0.5R$.

Figures 1 and 5, show that the estimation is just reasonable if no temperature measurement error exists, i.e., $\sigma = 0.0$. The estimation for the space and time-varying heat transfer coefficients, is reasonably for $\sigma = 0.0$ and 0.01, while for larger errors, $\sigma = 0.02$ and 0.05, the estimation is too far from the data computed by the direct numerical formulation.

CONCLUSION

We solved an inverse heat conduction problem for the estimation of the time-varying heat transfer coefficients as that encountered in the probloem of a cold jet impinging onto a heated surface. The reference data was obtained by a numerical direct simulation of an existing experimental rig.

We have shown that the estimated heat transfer coefficients agree quite well with the "exact" heat transfer coefficients, for experimental errors up to $\sigma = 0.02$. For values of σ smaller than 0.01 the heat transfer coefficients could be estimated quite reasonably. Otherwise, the estimation disagrees from the "exact" heat transfer coefficient. The data acquisition rate has no significant effect on the estimation of the time-varying heat transfer coefficient.

Therefore, heat transfer coefficients for the flow over a heated plate can be accurately estimated by solving an one-dimensional inverse heat conduction problem based on computed temperature data on the top surface of the plate. Please note that there are some physical limitations on the time scale of the heat transfer coefficient variation that can be accurately estimated by the proposed inverse analysis. Near the impingement region, $r < 0.1$, the estimated heat transfer coefficient differs considerably from the "exact" distribution in all cases. Thus, the inverse solution cannot capture the correct amplitude of the variation of the heat transfer coefficients.

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